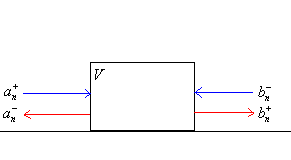
**Scattering/Transfer matrix formulation of scattering**

We’ve done this in quantum mechanics and I’d like to consider how much carries over into classical mechanics.

**Definition of the scattering matrix**

I think the first thing I would do is replace wavefunctions with currents. So to begin let’s examine the typical set up. We have a potential, V, represented by the box. This time, we send in currents from both sides – depicted by the blue lines. The incoming currents can have, in principle, a variety of longitudinal momenta kn. However the amplitudes are normalized so that each momenta (channel) carries unit flux.



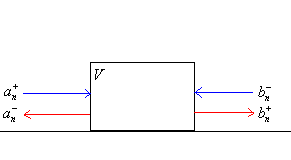


where **S** is the so-called scattering matrix and a-, a+, b-, b+ are each 2N×1 dimensional column vectors describing the component wavefunctions. From the equation, **S** is seen to relate the outgoing flux to the incoming flux. We can define another matrix, the transfer matrix **M**, which relates the wavefunction on the left to the one to the right. It is therefore defined as:



**S and M-matrices in terms of transmission and reflection coefficients**

Let’s write these matrices in terms of the usual transmission and reflection coefficients. Let’s start with the S-matrix.



From the picture above, let’s call r the probability that the current a+ will reflect, and the t the probability that it will transmit. And let’s call r′ the probability that the current b- will reflect, and t′ the probability that it will transmit. These are the transmission and reflection coefficients (matrices for more elaborate problems). Then as you can verify we would have the relationship between the probabilities:



which can be put into matrix form,



And so we can write the S-matrix as:



Now let’s write the transmission matrix in terms of these coefficients. Now rearrange to get the out/in coefficients:



Now we want to invert these relations and solve for the b’s, again allowing that they are spinors. Dividing by t′ in the first equation we have:



and plugging this into the second equation we get:



Putting this in matrix form we have:



and so we see that M can be written as:



And now we’ll relate to the usual transmission and reflection coefficients, T and R. So taking the usual approach where we have b- = 0. In that case, solving for the remaining coefficients in terms of the transmission/reflection coefficients, we get:



**Current conservation**

What are consequences?



Which can be related to S via:



And so we might presume



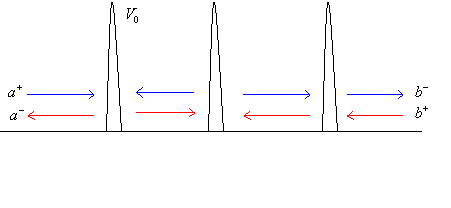
Which requires:



Which is as expected.

**Example: repeated δ function potential**

What if we repeat ?



Then how does the transmission eigenvalue change; how do the amplitudes change? Well first, for two δ’s.



and so the order of the transfer matrices sort of follows from right to left. Continuing this process, for N transfer matrices, we can see that we’d get:



Assuming parity symmetry, then we have:



Not sure I’ll be able to figure out general repeating pattern, but…



That’s encouraging. One more:



Alrighty. So then,



So the overal transmission and reflection coefficients are



Interestingly, the transmission probability decays as 1/n, but quantum mechanically, it would decay according to 1/n2. Perhaps this is due to phase coherence, and the path integral thing calculated before. Next, what are the two currents after the mth scatterer? Well we have:



The ratios are:



And we can see that this ratio continually decreases. So as the particles progress, they become less and less ‘disordered’.